Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

15 May 2026

Zone A morning | **Zone B** morning | **Zone C** morning

Session number

2 hours

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2226 – 5026

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Eight students run a 400-metre race. Their race time, T, and the length, L, of their legs are shown in the table.

Leg length (L cm)	85	82	78	91	76	84	80	88
Race time (T seconds)	65.2	67.3	70.0	60.8	72.5	66.1	68.9	62.4

The equation of the regression line of T on L for this data can be written in the form $T = aL + b$.

(a) Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use the equation of the regression line of T on L to predict the time of a student with a leg length of $79\,\text{cm}$. [2]

The equation of the regression line of L on T for this data can be written in the form $L = pT + q$.

(d) Find the value of p and the value of q. [1]

(This question continues on the following page)

(Question 1 continued)

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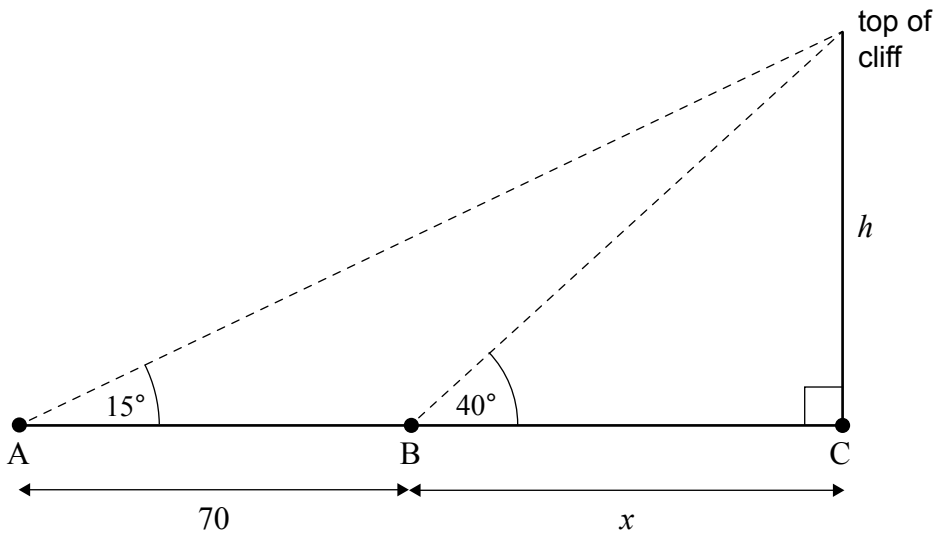
2. [Maximum mark: 6]

A ship is sailing directly towards a vertical cliff of height h metres.

diagram not to scale



When the ship is at point A, the angle of elevation to the top of the cliff is $15°$. After sailing a further 70 metres towards the cliff, the ship reaches point B, and the angle of elevation to the top of the cliff is $40°$.

Point C is at the base of the cliff, and $\mathrm{BC} = x$ metres.

(a) Find an expression in terms of h and x for

(i) $\tan 40°$;

(ii) $\tan 15°$. [2]

(b) Hence, find the value of x. [2]

(c) Find the value of h. [2]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 5]

The volume of air inside a balloon, V, measured in cubic metres, after t minutes is given by

$V = 5 + 20\mathrm{e}^{-kt}$, where $t \geq 0$ and k is a positive constant.

(a) Write down the initial volume of air inside the balloon. [1]

After 4 minutes, the volume of air inside the balloon is half the initial volume.

(b) Find the value of k. [2]

(c) Find the time at which the volume of air inside the balloon is 8 cubic metres. [2]

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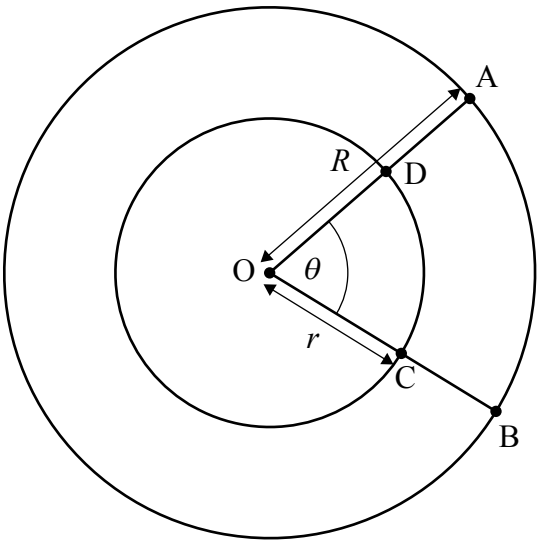
4. [Maximum mark: 7]

Two circles have a common centre, O. Points A and B lie on the circumference of the larger circle, which has radius $\mathrm{OA} = R$. Points C and D lie on $[\mathrm{OB}]$ and $[\mathrm{OA}]$ respectively, and they also lie on the circumference of the smaller circle. The smaller circle has radius $\mathrm{OC} = r$ and the acute angle $\mathrm{D\hat{O}C} = \mathrm{A\hat{O}B} = \theta$ radians.

This is shown in the following diagram.

diagram not to scale



The area of the sector OAB is $50\,\mathrm{cm}^2$ and its perimeter is $40\,\mathrm{cm}$. The area of the sector ODC is $18\,\mathrm{cm}^2$.

(a) Find the value of θ. [5]

(b) Hence, find the value of r. [2]

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5. [Maximum mark: 7]

A particle moves along a straight line with an initial displacement at O. Its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by

$$v(t) = (3t^2 - 10t + 4)\mathrm{e}^{-0.5t} - 4\cos t, \text{ where } 0 \leq t \leq 6.$$

(a) Find the maximum displacement of the particle. [4]

(b) Determine the intervals of time when the acceleration of the particle is negative. [3]

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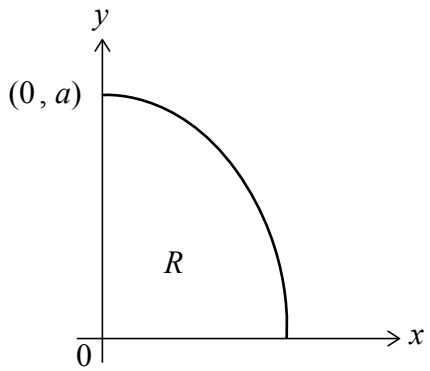
6. [Maximum mark: 6]

The following graph shows the curve $y = \sqrt{9\left(1 - \frac{x^2}{4}\right)}$ for $0 \leq x \leq 2$.

The curve intersects the y-axis at the point $(0, a)$.

diagram not to scale



(a) Write down the value of a. [1]

The region R is bounded by the curve, the x-axis and the y-axis. R is rotated through $360°$ about the y-axis to form a solid.

(b) Find the volume of this solid. [5]

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7. [Maximum mark: 7]

The continuous random variable X has a probability density function f, given by

$$f(x) = \begin{cases} \frac{3}{4}x^2(2-x) & 0 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\mathrm{P}(0.5 \le X < 1.5)$. [2]

The discrete random variable Y is defined as the continuous random variable X rounded to the nearest integer. Hence, Y can only take the value 0, 1 or 2.

The probability distribution for Y is given in the following table.

y	0	1	2
$\mathbf{P}(Y = y)$	a	b	c

(b) (i) Write down the value of b.

(ii) Find the value of a and the value of c. [3]

(c) Find $\mathrm{E}(Y)$. [2]

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8. [Maximum mark: 7]

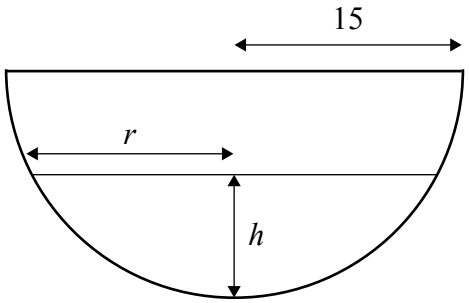
A bowl in the shape of a hemisphere with radius $15\,\text{cm}$ is being filled with water.

The water in the bowl has depth $h\,\text{cm}$ and surface radius $r\,\text{cm}$.

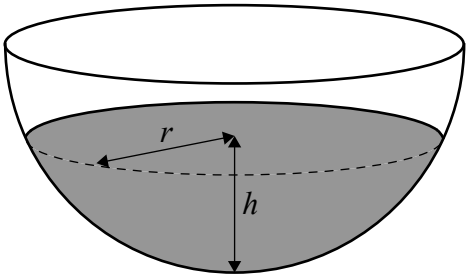
This information is shown in the following diagrams.

diagrams not to scale

2D diagram



3D diagram



(a) Show that $r = \sqrt{30h - h^2}$. [2]

Water is poured into the bowl at a constant rate of $20\,\text{cm}^3$ per minute.

The volume of water in the bowl is given by $V = \frac{\pi}{3}\left(45h^2 - h^3\right)$.

(b) Find the rate of change of r when $h = 5\,\text{cm}$. [5]

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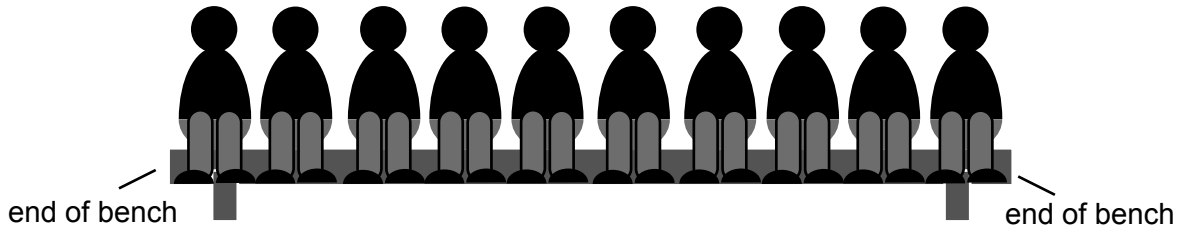
9. [Maximum mark: 6]

Seven girls and three boys sit on a bench in a single row for a group photo.

Determine the number of possible arrangements of the children, given that

(a) no boy sits next to any other boy; [3]

(b) no boy sits next to any other boy **and** no girl sits at an end of the bench. [3]



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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 18]

The waiting time, T minutes, of patients at a clinic can be modelled by a normal distribution with mean μ and standard deviation σ. The waiting times of all patients are independent of each other.

3.8% of the patients have a waiting time of less than 20 minutes, while 7.3% of the patients have a waiting time of more than 30 minutes.

(a) (i) Find the value of μ and the value of σ.

(ii) Hence, find the probability that a patient chosen at random has a waiting time of between 15 and 22 minutes. [7]

The doctor at the clinic only sees patients with an appointment.

Records show that 82.6% of patients attend their appointment.

The probability of a patient attending their appointment is independent of any other patient.

(b) On a particular day, 20 patients each made an appointment to see the doctor.

(i) Find the probability that at least 13 of these patients attended their appointment.

(ii) Given that at least 13 of these patients attended their appointment, find the probability that more than 2 patients did not attend their appointment. [7]

On any day, the doctor can see a maximum of 40 patients.

Some patients do not attend their appointments. The clinic decides to allow more than 40 appointments to be made each day.

The clinic aims for a probability of at least 99% that the doctor sees all the patients who arrive for their appointment.

(c) Find the greatest number of appointments that can be made to achieve the clinic's aim. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

The function f is defined by

$$f(x) = \frac{x^2 + bx + c}{x + d},$$

where $x \in \mathbb{R}$, $x \neq -d$ and $b, c, d \in \mathbb{Z}$.

The graph of $y = f(x)$ has asymptotes $x = 3$ and $y = x + 7$.

(a) Write down the value of d. [1]

(b) Show that $b = 4$. [2]

The graph of $y = f(x)$ has a local maximum at $x = -3$.

(c) Find the value of c. [4]

(d) Sketch the graph of $y = f(x)$ for $-30 \leq x \leq 30$. Show the values of any axis intercepts and the coordinates of any local maxima and minima, and label the asymptotes. [4]

The equation $|f(x)| = k$ has four solutions.

(e) Find the range of possible values of k. [2]

The graph of $y = g(x)$ is obtained from the graph of $y = f(x)$ by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(f) Find the equations of the vertical asymptotes of $y = \frac{1}{g(x)}$. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = y^3 - \frac{y}{x}$, for $x, y > 0$. It is given that $y = 2$ when $x = 1$.

(a) Use Euler's method with a step length of 0.01 to find an approximate value of y when $x = 1.05$. [3]

(b) By using the substitution $\frac{1}{y^2} = -2z$, show that the differential equation can be expressed as $\frac{\mathrm{d}z}{\mathrm{d}x} - \frac{2z}{x} = 1$. [4]

(c) Hence, solve the differential equation, giving your answer in the form $y = f(x)$. [7]

(d) Find the domain and range of $f(x)$. [3]

(e) By considering the shape of the graph of $f(x)$, explain whether Euler's method gives an overestimate or underestimate of the value of y when $x = 1.05$. [1]

Please **do not** write on this page.

Answers written on this page
will not be marked.